APPROXIMATE CALCULATION OF THERMOELASTIC STRESSES UNDER NONSTATIONARY CONDITIONS

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An analytical method of calculating thermal stresses for mixed boundary conditions of the second and third kind is proposed. Problems are solved for arbitrary time-variations of the temperature of the ambient medium.

An effective approximate method of calculating thermal stresses under nonstationary conditions is proposed, which even in the first approximation is more accurate than the solutions obtained in [4]. The determination of the temperature field is based on the use of integral transforms in combination with variational methods [5, 8].

Plane Wall. The temperature distribution in an unbounded plate of thickness R for mixed boundary conditions of the second and third kind

$$T(x,\ 0)=T_0,\quad \left(-\lambda\frac{\partial T}{\partial x}\right)_{x=0}=\varphi_1(t),\quad \left\{\frac{\partial T}{\partial x}=h\left[\varphi_2(t)-T(x,\ t)\right]\right\}_{x=R} \tag{1}$$

in the Laplace transform domain, is sought among functions of the form

$$T_n^*(\xi,\ p)=\varphi_2^*(p)+\frac{\varphi_1^*(p)R}{\lambda}\left(\xi-\frac{\mathrm{Bi}+1}{\mathrm{Bi}}\right)+a_1^*(p)\left(\xi^2-\frac{\mathrm{Bi}+2}{\mathrm{Bi}}\right)+\sum_{k=1}^{n}a_{k+1}^*(p)\,\xi^{2(k-1)}(1-\xi^2)^2. \tag{2}$$

The coefficients $a_1^*(p)$, $a_2^*(p)$, ..., $a_n^*(p)$ for which expression (2) satisfies best the heat-conduction equation are determined by the Bubnov–Galerkin method [5, 8]. After calculating these coefficients and passing to the domain of inverse transforms, we obtain the approximate temperature field.

In the first approximation, for arbitrary boundary conditions, the temperature can be written in the form

$$T(\xi,\ \mathrm{Fo},\ \mathrm{Bi})=\varphi_2(\mathrm{Fo})+\frac{\varphi_1(\mathrm{Fo})R}{\lambda}\left(\xi-\frac{\mathrm{Bi}+1}{\mathrm{Bi}}\right)+a_1(\mathrm{Fo})\left(\xi^2-\frac{\mathrm{Bi}+2}{\mathrm{Bi}}\right), \tag{3}$$

where

$$a_1(\mathrm{Fo})=\frac{D(\mathrm{Bi})}{2}\int\limits_0^{\mathrm{Fo}}\Phi_2(\mathrm{Fo}^*)\exp\left[-D(\mathrm{Bi})(\mathrm{Fo}-\mathrm{Fo}^*)\right]d\,\mathrm{Fo}^*$$

$$-C(\mathrm{Bi})\left\{\frac{\varphi_1(\mathrm{Fo})R}{\lambda}-\frac{D(\mathrm{Bi})a}{\lambda R}\int\limits_0^{\mathrm{Fo}}\varphi_1(\mathrm{Fo}^*)\exp\left[-D(\mathrm{Bi})(\mathrm{Fo}-\mathrm{Fo}^*)\right]d\,\mathrm{Fo}^*;\right. \tag{4}$$

$\Phi_2(\mathrm{Fo})$ is the inverse transform of $p\varphi_2^*(p)-T_0$; $\xi=x/R$;

$$D(\mathrm{Bi})=\frac{5\mathrm{Bi}(\mathrm{Bi}+3)}{2\mathrm{Bi}^2+10\mathrm{Bi}+15};\quad C(\mathrm{Bi})=\frac{5(5\mathrm{Bi}^2+20\mathrm{Bi}+24)}{16(2\mathrm{Bi}^2+10\mathrm{Bi}+15)}.$$

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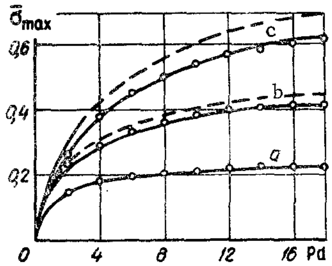


Fig. 1

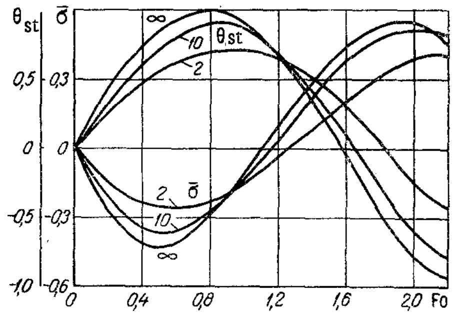


Fig. 2

Fig. 1. Maximal thermoelastic stresses at the plate surface for Bi = 1 (a), 4 (b), ∞ (c). Solid curves correspond to exact calculations, dashed curves to data [4], points to formula (9).

Fig. 2. Changes in the relative temperature θ (1, Fo) and stress $\bar{\sigma}$ at the plate surface in the case of harmonic fluctuations of the ambient temperature (Pd = 2). Numbers at the curves denote the values of the Bi number.

The temperature calculated from formula (3) for $\varphi_1(t) = -q = \text{const}$, $\varphi_2(t) = T_c = T_0$ is in satisfactory agreement with the exact solution for Fo ≥ 0.1 [8].

Since the thermal stresses at the surface of an element situated in a fluid (heat-transfer agent) may be normally considered as the greatest threat, we shall calculate the tangential stresses at the plate surface.

According to [3], the thermoelastic surface stresses for a plate can be determined from the formula

$$\bar{\sigma} = \frac{\alpha_T E}{1-\nu}\left[\bar{T}(\mathrm{Fo}) - T_{st}\right], \tag{5}$$

where

$$\bar{T}(\mathrm{Fo}) = \int_0^1 T(\xi, \mathrm{Fo})\, d\xi.$$

For an exponential drop in the temperatures of the medium and the thermal insulation of the other wall

$$\varphi_1(t) = 0, \quad \varphi_2(t) = T_0 - \Delta T\left[1 - \exp(-mt)\right] = T_0 - \Delta T\left[1 - \exp(-\mathrm{PdFo})\right] \tag{6}$$

from solution (3), we obtain

$$T(\xi, \mathrm{Bi}, \mathrm{Fo}, \mathrm{Pd}) = T_0 - \Delta T\left[1 - \exp(-\mathrm{Pd\,Fo})\right]$$
$$+\frac{D(\mathrm{Bi})\,\mathrm{Pd}\,\Delta T}{2\left[D(\mathrm{Bi}) - \mathrm{Pd}\right]}\left\{\exp(-\mathrm{Pd\,Fo}) - \exp\left[-D(\mathrm{Bi})\,\mathrm{Fo}\right]\right\}\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right). \tag{7}$$

The relative stress can be reduced to the form

$$\bar{\sigma} = \frac{\sigma(1-\nu)}{\Delta T E \alpha_T} = \frac{D(\mathrm{Bi})\,\mathrm{Pd}}{3\left[D(\mathrm{Bi}) - \mathrm{Pd}\right]}\left\{\exp(-\mathrm{PdFo}) - \exp\left[-D(\mathrm{Bi})\,\mathrm{Fo}\right]\right\}. \tag{8}$$

By differentiating with respect to Fo and analyzing for maxima, we obtain

$$\bar{\sigma}_{\max} = \frac{D(\mathrm{Bi})}{3}\left(\frac{D}{\mathrm{Pd}}\right)^{-\frac{D}{D-\mathrm{Pd}}}, \tag{9}$$

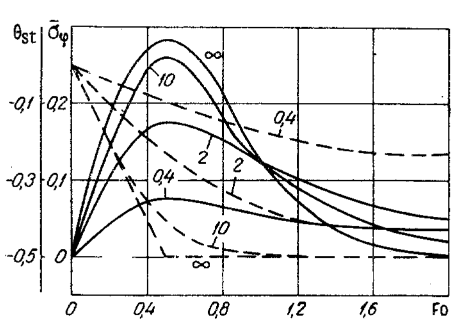
to which corresponds the dimensionless time

$$\mathrm{Fo}_* = \left[\ln D(\mathrm{Bi}) - \ln \mathrm{Pd}\right]\left[D(\mathrm{Bi}) - \mathrm{Pd}\right]^{-1}. \tag{10}$$

Solutions in a form similar to (8)-(10) were obtained in [4], with the only difference that the quantity

$$\varphi(\mathrm{Bi}) = \frac{3\mathrm{Bi}}{3+\mathrm{Bi}}$$

was substituted for D(Bi).



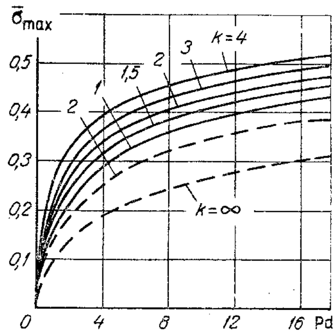


Fig. 3 Fig. 4

Fig. 3. Changes in the relative temperature θ_{st} and stress $\bar{\sigma}_{\varphi}$ at the inside surface of the tube for a linear drop in the temperature of the medium over a finite interval of time $\mathrm{Fo}_{\tau} = 0.5$. Solid lines correspond to $\bar{\sigma}_{\varphi}$, dashed lines to θ_{st}. The numbers at the curves are values of Bi.

Fig. 4. Value of σ_{max} at a noninsulated tube surface for Bi = 4, the temperature drop of the medium according to an exponential law. Solid lines correspond to calculations for the fluid flow inside the tube, dashed lines to calculations for external heat transfer.

Figure 1 shows the stresses calculated from formula (9) for Bi = 1, 4, ∞, and Pd numbers ranging from 0 to 20. The figure also shows the relations $\bar{\sigma}_{max} = f(\mathrm{Bi}, \mathrm{Pd})$ obtained from an exact solution and from an approximate formula [4]. The comparison demonstrates the high accuracy of the method proposed. Similar results are obtained when Fo_* is determined from formula (10).

In heat-conduction theory it is known that the time-variation of the temperature field in a plate for boundary conditions of the third kind is described under regular conditions by an exponential function with the exponent $\mu_1^2\mathrm{Fo}$, where μ_1 is the first (smallest) root of the equation

$$\operatorname{ctg}\mu = \frac{\mu}{\mathrm{Bi}}.$$

In the approximate solution (7), the rate of stabilization is D(Bi). When the Bi number varies from 0 to 1, the difference between D(Bi) and μ_1^2 does not exceed 0.08%. For $1 \le \mathrm{Bi} \le 10$, the value of D(Bi) is greater than that of μ_1^2 by not more than 1%, and for $10 \le \mathrm{Bi} \le \infty$, by not more than 1.3%. Consequently, one may propose the following computational formula

$$m_0 = \mu_1^2 = \frac{5(\mathrm{Bi}^2 + 3\mathrm{Bi})}{2\mathrm{Bi}^2 + 10\mathrm{Bi} + 15}. \tag{11}$$

This expression provides an analytical relationship between the coefficient m_0 (rate of cooling) and the Bi number for regular conditions [1].

In [4], the rate of cooling is expressed by $\varphi(\mathrm{Bi})$. For $1 \le \mathrm{Bi} \le 10$, the value of $\varphi(\mathrm{Bi})$ exceeds that of μ_1^2 by 13%. For Bi = ∞, the maximum error is 21%, i.e., with increasing Bi number, the function $\varphi(\mathrm{Bi})$ deviates appreciably from the value of μ_1^2. At the same time, the maximum error of D(Bi) for Bi = ∞ is merely 1.3%. Thus, replacing $\varphi(\mathrm{Bi})$ by the function D(Bi) would improve the accuracy of the formulas proposed in [4] for calculating thermal stresses.

For harmonic temperature fluctuations of the ambient medium and the thermal insulation of the other wall

$$\varphi_1(t) = 0,\ \ \varphi_2(t) = T_0 + \Delta T \sin\omega t = T_0 + \Delta T \sin(\mathrm{Pd\,Fo}) \tag{12}$$

solution (3) reduces to the form

$$T(\xi,\ \mathrm{Fo},\ \mathrm{Bi},\ \mathrm{Pd}) = T_0 + \Delta T \sin(\mathrm{Pd\,Fo})$$

$$+\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right)\frac{\Delta T\, D(\mathrm{Bi})\cos\varphi_0}{2}\left\{\sin\varphi_0 \exp\left[-D(\mathrm{Bi})\,\mathrm{Fo}\right] - \sin(\mathrm{Pd\,Fo} + \varphi_0)\right\}, \tag{13}$$

where

$$\sin\varphi_0 = \frac{m}{\sqrt{\omega^2+m^2}}, \quad \cos\varphi_0 = \frac{\omega}{\sqrt{\omega^2+m^2}}, \quad \mathrm{Pd} = \frac{\omega R^2}{a}, \quad m = \frac{D(\mathrm{Bi})\,a}{R^2}.$$

The thermoelastic surface stress of a plate heated by the ambient medium by a harmonic law is determined from the formula

$$\bar{\sigma} = \frac{\sigma(1-\nu)}{E\alpha_T\Delta T} = \frac{D(\mathrm{Bi})\cos\varphi_0}{3}\{\sin\varphi_0\exp[-D(\mathrm{Bi})\,\mathrm{Fo}] - \sin(\mathrm{PdFo}+\varphi_0)\}. \tag{14}$$

By differentiating (14) with respect to the parameter Fo and equating to zero, we obtain

$$D(\mathrm{Bi})\exp[-D(\mathrm{Bi})\,\mathrm{Fo}] = -\frac{\mathrm{Pd}}{\sin\varphi_0}\cos(\mathrm{PdFo}+\varphi_0). \tag{15}$$

By solving graphically the transcendental equation (15) for given values of Bi and Pd, we obtain the moment of time Fo_* to which corresponds the maximum stress. Figure 2 shows the changes in the relative temperature $\theta(1, \mathrm{Fo}, \mathrm{Bi}, \mathrm{Pd}) = (T(1, \mathrm{Fo}) - T_0)/\Delta T$ and in the stress $\bar{\sigma}$ calculated from formulas (13) and (14) for Pd = 2 and Bi = 2, 10, ∞. The thermal stresses and the temperature changes over the entire thickness of the plate can be analyzed on the basis of solution (13).

It is noteworthy that solutions of such problems for the boundary conditions (12) find practical applications in the study of thermal stresses in large concrete blocks (for example in dams) exposed to diurnal variations of the air temperature [6].

Solid Cylinder. The radial and tangential stresses are determined from formulas [2]

$$\sigma_r = \frac{E\alpha_T}{2(1-\nu)}[\bar{T}(R, t) - \bar{T}(r, t)], \tag{16}$$

$$\sigma_\varphi = \frac{E\alpha_T}{2(1-\nu)}[\bar{T}(R, t) + \bar{T}(r, t) - 2T(r, t)], \tag{17}$$

where

$$\bar{T}(r, t) = \frac{2}{r^2}\int_0^r T(\rho, t)\,\rho\,d\rho.$$

In transforms, the temperature distribution in the first approximation for boundary conditions of the third kind are defined by the formula

$$T^*\left(\frac{r}{R}, p\right) = \varphi^*(p) + \frac{A(\mathrm{Bi})}{4}\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \left(\frac{r}{R}\right)^2\right][T_0 - p\varphi^*(p)]\left[p + \frac{A(\mathrm{Bi})\,a}{R^2}\right]^{-1}, \tag{18}$$

where

$$A(\mathrm{Bi}) = \frac{6(\mathrm{Bi}^2+4\mathrm{Bi})}{\mathrm{Bi}^2+6\mathrm{Bi}+12}. \tag{19}$$

By changing to the inverse transform domain, we obtain the temperature distribution in a cylinder for any law of ambient temperature variation $\varphi(t)$. Specifically, for $\varphi(t) = T_c$ = const, the relative temperature is described by the formula

$$\theta\left(\frac{r}{R}, \mathrm{Fo}, \mathrm{Bi}\right) = \frac{T(r, t)-T_0}{T_c-T_0} = 1 - 0.25A(\mathrm{Bi})\exp[-A(\mathrm{Bi})\,\mathrm{Fo}]\left[\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \left(\frac{r}{R}\right)^2\right]. \tag{20}$$

Let us determine $\bar{T}(R, t)$, $\bar{T}(r, t)$, $T(r, t)$ from formula (20), and substitute the values obtained into (16), (17). Then,

$$\bar{\sigma}_r = \frac{\sigma_r(1-\nu)}{E\alpha_T(T_c-T_0)} = \frac{A(\mathrm{Bi})}{16}\left[1-\left(\frac{r}{R}\right)^2\right]\exp[-A(\mathrm{Bi})\,\mathrm{Fo}], \tag{21}$$

$$\bar{\sigma}_\varphi = -\frac{\sigma_\varphi(1-\nu)}{E\alpha_T(T_c-T_0)} = \frac{A(\mathrm{Bi})}{16}\left[1-3\left(\frac{r}{R}\right)^2\right]\exp[-A(\mathrm{Bi})\,\mathrm{Fo}]. \tag{22}$$

TABLE 1. Values of μ_1^2

$k=\frac{R_2}{R_1}$		Bi 0,5	1	3	5	7	10
$k=1,25$	according to [7]	1,7056	3,2761	8,4797	12,3693	—	—
	from (28)	1,7065	3,2800	8,4798	12,3694	15,3613	18,7200
$k=1,5$	according to [7]	0,7259	1,3710	3,1577	4,2477	4,9729	5,6836
	from (28)	0,7380	1,3712	3,1610	4,2510	4,9750	5,6905

For $0.06 \leq \mathrm{Fo} \leq 0.1$, expressions (21), (22) yield results which deviate from the exact solution by not more than 8-10%, while for $\mathrm{Fo} \geq 0.1$ the results are almost identical with the exact solutions.

It is noteworthy that the function A (Bi) defined by formula (19) approximates satisfactorily the square of the first (smallest) root of the equation

$$\frac{J_0(\mu)}{J_1(\mu)} = \frac{\mu}{\mathrm{Bi}}.$$

Stresses in the Tubes (Hollow Cylinders). For a thermally insulated outside surface, $\varphi_1(t) = 0$, and a temperature of the medium inside the tube varying according to the law

$$\varphi_2(t) = T_0 - \Delta T t + \Delta T(t-\tau)H(t-\tau) = T_0 - \mathrm{Pd}^*(\mathrm{Fo} - \mathrm{Fo}_\tau)H(\mathrm{Fo} - \mathrm{Fo}_\tau) \tag{23}$$

the temperature distribution over the wall thickness can be reduced to the form

$$\begin{aligned} T(r, \mathrm{Fo}, \mathrm{Bi}, \mathrm{Fo}_\tau) &= T_0 - \mathrm{Pd}^*\mathrm{Fo} + \mathrm{Pd}^*(\mathrm{Fo} - \mathrm{Fo}_\tau)H(\mathrm{Fo} - \mathrm{Fo}_\tau) \\ &+ \mathrm{Pd}^* D(\mathrm{Bi}, k)\{[1 - \exp(-A(\mathrm{Bi}, k)\mathrm{Fo})] \\ &- H(\mathrm{Fo} - \mathrm{Fo}_\tau)[1 - \exp(-A(\mathrm{Bi}, k)(\mathrm{Fo} - \mathrm{Fo}_\tau))]\}\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right), \end{aligned} \tag{24}$$

where

$$\begin{aligned} D(\mathrm{Bi}, k) &= \frac{\mathrm{Bi}(5k+3) + 12(k+1)}{4[\mathrm{Bi}(k+3) + 12]}; \\ A(\mathrm{Bi}, k) &= \frac{10\mathrm{Bi}[\mathrm{Bi}(k+3) + 12]}{\mathrm{Bi}^2(11k+5) + 10\mathrm{Bi}(5k+3) + 60(k+1)}. \end{aligned} \tag{25}$$

In these expressions $k = R_2/R_1$; $H(\mathrm{Fo} - \mathrm{Fo}_\tau)$ is the Heaviside function ($H(\mathrm{Fo} - \mathrm{Fo}_\tau) = 0$ for $\mathrm{Fo} < \mathrm{Fo}_\tau$, $H(\mathrm{Fo} - \mathrm{Fo}_\tau) = 1$ for $\mathrm{Fo} \geq \mathrm{Fo}_\tau$); $\xi = (r - R_1)/\Delta R$; $\mathrm{Pd}^* = \Delta T(\Delta R)^2/a$; $\mathrm{Fo}_\tau = a\tau/(\Delta R)^2$; $\Delta R = R_2 - R_1$. The temperature drops linearly during a time τ, and then remains constant and equal to $T_0 + \Delta T\tau$.

The tangential stress at the outside surface of the tube is

$$\begin{aligned} \bar{\sigma}_\varphi(R_1) &= \frac{\sigma_\varphi(R_1)(1-\nu)}{E\alpha_T \mathrm{Pd}^*} = \frac{D(\mathrm{Bi}, k)(5k+3)}{6(k+1)} \\ &\times \{[1 - \exp(-A\mathrm{Fo})] - H(\mathrm{Fo} - \mathrm{Fo}_\tau)[1 - \exp(-A(\mathrm{Fo} - \mathrm{Fo}_\tau))]\}. \end{aligned} \tag{26}$$

Figure 3 shows changes in the relative temperature $\theta = (T - T_0)/\mathrm{Pd}^*$ and the stresses $\bar{\sigma}_\varphi(R_1)$ at the inside surface of the tube for $\mathrm{Fo}_\tau = 0.5$, $\mathrm{Bi} = 0.4, 2, 10, \infty$, and $k = 2$.

Solution of the boundary-value problem for nonstationary heat conduction in a hollow cylinder for mixed boundary conditions of the second and third kind with the aid of Grinberg–Koshlakov's integral transforms leads to the solution of the Sturm–Liouville problem, the eigenvalues of which are determined from the solution of the transcendental equation

$$\frac{V_0(\mu_n)}{V_1(\mu_n)} = -\frac{\mu_n}{\mathrm{Bi}}, \tag{27}$$

where

$$V_0(\mu_n) = Y_1(\mu_n k) J_0(\mu_n) - J_1(\mu_n k) Y_0(\mu_n),$$
$$V_1(\mu_n) = Y_1(\mu_n k) J_1(\mu_n) - J_1(\mu_n k) Y_1(\mu_n).$$

The method proposed in this paper for calculating nonstationary heat conduction implies that expression A (Bi, k) must yield the functional dependence of the square of the first root of Eq. (27 on the Bi number and the parameter k. We refer the Bi number and Fo to the inside radius Bi = $\alpha R_1/\lambda$, Fo = at/R_1^2. The expression A (Bi, k) reduces then to the form

$$A^*(\mathrm{Bi},\ k) = \frac{10\mathrm{Bi}\,(k-1)^{-1}\,[\mathrm{Bi}\,(k-1)\,(k+3)+12]}{\mathrm{Bi}^2\,(k-1)^2\,(11k+5)+10\mathrm{Bi}\,(k-1)\,(5k+3)+60\,(k+1)}. \tag{28}$$

Values of μ_1 calculated to the third decimal digit for Bi = 0.5, 1.3, 10, and k = 1.25, 1.50 are given in [7]. Assuming them to be exact, we compare our calculations from formula (28). The results of the calculations and of the comparison with the exact values are compiled in Table 1. The results are in satisfactory agreement.

In order to determine the second root μ_2, one must obtain a solution in the second approximation. We obtain then, at the same time, an improved formula for μ_1^2. Our calculations showed that, in the second approximation, the value of μ_1 conforms with the exact value within an accuracy to the third decimal digit.

For a temperature drop of the medium according to the power law (6) and of the thermal insulation of the outside surface, the tangential stress at the inside surface of the tube reduces to the form

$$\bar{\sigma} = \frac{\sigma_\varphi(1-\nu)}{\alpha_T E\,\Delta T} = \frac{A_0(\mathrm{Bi},\ k)\,\mathrm{Pd}\,(5k+3)}{6\,[A(\mathrm{Bi},\ k)-\mathrm{Pd}]\,(k+1)}\,\{\exp(-\mathrm{PdFo})-\exp[-A(\mathrm{Bi},\ k)\,\mathrm{Fo}]\}. \tag{29}$$

The maximum stress is obtained from formula

$$\bar{\sigma}_{\max} = \frac{A_0(\mathrm{Bi},\ k)\,\mathrm{Pd}\,(5k+3)}{6\,(k+1)}\left[\frac{A(\mathrm{Bi},\ k)}{\mathrm{Pd}}\right]^{-\frac{A(\mathrm{Bi},k)}{A(\mathrm{Bi},k)-\mathrm{Pd}}}, \tag{30}$$

where

$$A_0(\mathrm{Bi},\ k) = \frac{5\mathrm{Bi}\,[\mathrm{Bi}\,(3k+5)+12\,(k+1)]}{2\,[\mathrm{Bi}^2\,(5k+11)+10\mathrm{Bi}\,(3k+5)+60(k+1)]}.$$

The time to $\bar{\sigma}_{\max}$ is determined from formula (10), provided A (Bi, k) is substituted for D (Bi).

Figure 4 shows the changes in $\bar{\sigma}_{\max}$ calculated from formula (30) for Bi = 4, k = 1, 1.5, 2 with changes in the Pd number from 0 to 20. From the solution of (26)-(30) for k → 1, we obtain the corresponding formulas for a plate.

Finally, it should be noted that the stresses at the contact surface with the fluid for external flow and thermal insulation of the inside wall of the tube can be studied with the aid of formulas (26)-(30), provided in these solutions 1/k is substituted for k. The dashed lines in Fig. 4 correspond to $\bar{\sigma}_{\max}$ calculated for external contact between the tube (k = 2) and the fluid and for a solid cylinder (k = ∞, $R_1 \to 0$, $R_2 < \infty$).

Thus, the approximate method of calculating thermoelastic stresses, based on the use of integral transforms and variational methods of determining the temperature field inside the body under consideration, yields highly satisfactory results and at the same time appreciably reduces the computational labor.

NOTATION

σ is the stress;
T is the temperature of the body;
$\bar{T}$ is the mean temperature;
T^* is the representation of the temperature in Laplace transforms;
p is the Laplace transform parameter;
E is the modulus of elasticity;
ν is the Poisson ratio;
α_T is the linear expansion coefficient;
Bi = hR = $\alpha R/\lambda$, Fo = at/R^2, Pd = mR^2/a or Pd = $\omega R^2/a$ is Predvoditelev's number;
H(t − τ) is the unit Heaviside function;
$\mathrm{Fo}_\tau = a\tau/R^2$;
τ is the time of linear rise (drop) in the temperature of the medium.

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